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Model for the Mean Wind and Turbulence Within and Above an Urban Canopy

Part I: Theory and Numerical Implementation

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MODEL FOR THE MEAN WIND AND TURBULENCE WITHIN AND ABOVE AN URBAN CANOPY
PART I: THEORY AND NUMERICAL IMPLEMENTATION

by

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ABSTRACT

In this paper, a three-dimensional (3-D) model for the disturbed mean wind flow and turbulence in an urban canopy composed of an extensive array of obstacles is developed. The context of this work is the provision of a computational tool for the rapid diagnosis of transport and dispersion of toxic contaminants (e.g., chemical and biological warfare agents) over a relatively homogeneous urban area. A variant of the $K \propto \lambda k^{1/2}$ first-order closure (where K is the eddy viscosity, k is the turbulence kinetic energy (TKE) calculated from its simplified governing differential equation, and λ is an algebraically specified turbulence length scale) is studied and used to close the mean momentum equations and the TKE transport equation. The turbulence length scale λ has been designed to reproduce the known characteristics of the length scale in a canopy; namely, recognition of the limitation imposed on eddy size by proximity to the ground, the constant length scale across much of the upper canopy that is intimately connected to the canopy shear length scale (vorticity thickness), and the displacement of the equilibrium flow structure above the canopy due to the presence of the canopy. A method for the partitioning of the TKE into its component normal Reynolds stresses is obtained. The latter is based on the derivation of a particular algebraic stress model that has been derived from a stress transport model due to Launder, Reece and Rodi (Journal of Fluid Mechanics, **68**, pp. 537–566, 1975) that has been modified to include the extra physics arising from the interaction of the airflow with the obstacles. The effect of the obstacles on the flow is parameterized in terms of an effective drag parameter, thus foregoing the complications involved in the generation of a body-fitted grid about the obstacles and imposing no slip/no leak boundary conditions at the walls of the obstacles. Details of the numerical implementation of this model are provided. An example of a simulation of the disturbed mean flow in an urban array is given.

EXECUTIVE SUMMARY

Title: Model for the Mean Wind and Turbulence Within and Above an Urban Canopy: Part I: Theory and Numerical Implementation

Authors: Eugene Yee and John D. Wilson

Introduction: Incidents in Tokyo involving the use of chemical and biological warfare (CBW) agents by a terrorist organization have mobilized both military and public concerns on the use of these weapons against military and civilian populations. The nature of the threat has changed significantly in the 1990s with the potential for CBW covert operations and terrorist incidents as adjuncts to military operations becoming more and more likely each year. With the ever-changing nature of coalition force operations coupled with the expected expansion in the spectrum of the conflict, the Canadian Forces (CF) will more likely than ever be confronted with an expansion of the battle space, being called upon to operate beyond its traditional areas of interest. With the emergence of asymmetrical threats posed by the fragmented, worldwide proliferation of CBW technology and delivery capabilities, the threat of the use of CBW weapons against CF is likely to increase, particularly in deployments where CF are operating as part of a multi-national force required to fight in or protect urban areas, whether in battle, peacemaking, peacekeeping, or counter-terrorist operations. For an effective defence against the use of CBW agents in an urban environment, the CF will require an accurate urban dispersion model to track the agent release in the urban area, to assess the hazard, to predict the expected number of casualties and injuries and subsequent human performance degradation, and to develop the operational procedures and regulations required to control, mitigate, and monitor the fate of the CBW agent in the urban area. However, it should be emphasized that the science of turbulent diffusion in a built-up domain consisting of large groups of obstacles, which underpins these important practical problems, is still not understood to enough depth to allow (in general) well-founded, reliable, and practically useful models for urban dispersion to be constructed. While undeniable, this fact is undoubtedly surprising to most non-experts; nor, regrettably, has it inhibited many people from producing and selling software which, however attractively packaged, cannot fulfill the purposes for which it was purchased because it is based

on inadequate, often wrong, science.

Proper understanding of the turbulent dispersion of contaminants to and from urban canopies requires an understanding of the atmospheric turbulence within and above the canopy. Arguably, the most important scientific challenge facing the modelling of dispersion in an urban canopy is to improve our understanding of, and means to predict, the spatial variation of turbulent wind flow through a built-up domain. Indeed, there is an important reason for wishing to predict the nature of turbulent flow through an urban canopy on the basis of a few simple inputs (e g., a physical description of the urban canopy and values of wind-speed and direction at some reference height above the canopy). Prediction of urban canopy flow is pre-requisite to or co-requisite with prediction of problem of contaminant (CBW agent) dispersion within an urban canopy. Physically-based models using the Lagrangian stochastic theory of turbulent dispersion are enticingly simple in concept and can be used to calculate dispersion in an urban canopy, but these approaches require that mean flow and turbulence quantities (e.g., the velocity variances, the shear stresses, the dissipation of turbulence kinetic energy, etc.) be supplied as external inputs to the model.

Given the complexities in the atmospheric flow through groups of buildings and obstacles comprising an urban canopy, the purpose of the present study is to derive systematically a set of constitutive equations for computation of turbulent flows and extraction of the requisite turbulence quantities (mean velocity, velocity variances, shear stresses, turbulence kinetic energy, and dissipation rate) within and above an urban canopy of regular structures. The basic goal is to construct an urban flow model with a minimum amount of complexity that nevertheless captures the essence of the relevant physics of disturbed flows through a built-up domain that can be directly used to "drive" a generalized dispersion model for the prediction of urban dispersion.

Results: In this paper, a three-dimensional (3-D) model for the disturbed mean wind flow and turbulence in an urban canopy comprised of an extensive array of obstacles is developed. The context of this work is the provision of a computational tool for the rapid diagnosis of transport and dispersion of toxic contaminants (e.g., chemical and biological warfare agents) over a relatively homogeneous urban area. A variant of the $K \propto \lambda k^{1/2}$ first-order closure (where K is the eddy viscosity, k is the turbulence kinetic energy (TKE) calculated from its simplified governing differential equation, and λ is an algebraically specified turbulence

length scale) is studied and used to close the mean momentum equations and the TKE transport equation. The turbulence length scale λ has been designed to reproduce the known characteristics of the length scale in a canopy; namely, recognition of the limitation imposed on eddy size by proximity to the ground, the constant length scale across much of the upper canopy that is intimately connected to the canopy shear length scale (vorticity thickness), and the displacement of the equilibrium flow structure above the canopy due to the presence of the canopy. A method for the partitioning of the TKE into its component normal Reynolds stresses is obtained. The latter is based on the derivation of a particular algebraic stress model that has been derived from a stress transport model due to Launder, Reece and Rodi (Journal of Fluid Mechanics, **68**, pp. 537–566, 1975) that has been modified to include the extra physics arising from the interaction of the airflow with the obstacles. The effect of the obstacles on the flow is parameterized in terms of an effective drag parameter, thus foregoing the complications involved in the generation of a body-fitted grid about the obstacles and imposing no slip/no leak boundary conditions at the walls of the obstacles. Details of the numerical implementation of this model are provided. An example of a simulation of the disturbed mean flow in an urban array is given.

Significance of Results and Future Work: The development and numerical implementation of a 3-D model for the disturbed mean flow and turbulence in a regular urban canopy comprised of a large group of obstacles is the first stage in the development of a physically sound model for urban dispersion. The importance of the current work is that it provides a suitable fully 3-D wind field in an urban array that can be used to "drive" a generalized dispersion model to predict the transport and turbulent diffusion of toxic contaminants in the urban environment. A useful picture of the variation of the wind within a "unit cell" of the urban canopy is provided by the current model.

The critical issue facing the current model is validation. There will be a pressing need to deal with the great volume and specificity of the model outputs from the current program, and to take these predictions beyond mere qualitative statements of interpretation such as "looks reasonable!" to more critical quantitative tests of the predictions. What would that test or series of tests be? As already mentioned, we do not seek to capture arcane flow details on individual buildings, so there is no point to turn to experiments on flow about single obstacles. With regard to this point, we have developed the current model to provide

predictions of flow through large groups of buildings such as exist within a domain inside a large city. Unfortunately, we have not yet been able to identify an experimental study in which the flow within an extensive building array (viz., extensive enough to induce large-scale uniformity in the array and justify the assumption of periodicity inherent in the present model) is measured with comprehensive detail relative to separation length scales L_x and L_y. New laboratory experiments designed to provide these comprehensive flow details within an extensive urban canopy are currently being developed at Defence Research Establishment Suffield to obtain the necessary information to validate the current model. In the meantime, one might perhaps test the present model against the observations of Raupach, Coppin, and Legg (Boundary-Layer Meteorology, **35**, pp. 21–52, 1986) who comprehensively describe a wind tunnel simulation with a very long fetch over a staggered array of very thin obstacles (i.e., vertical bars or "tombstones"). However, this unusual canopy was designed more as an archetype for a plant canopy rather than an urban canopy.

Presently, a full 3-D computation of the disturbed flow field in an urban array requires computation times of the order of hours on a PC. In order for the model to be useful for operational applications, efficient algorithms will need to be developed that would enable numerical solutions for the coupled partial differential equations describing mean momentum and TKE transport to be obtained in computational times of the order of minutes. Certainly, with improvements in algorithms as well as in computer performance (CPU speeds, etc.), it is anticipated that this objective can be achieved within the next two years.

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I. INTRODUCTION

Incidents in Tokyo involving the use of chemical and biological warfare (CBW) agents by a terrorist organization have mobilized both military and public concerns on the use of these weapons against military and civilian populations. The nature of the threat has changed significantly in the 1990s with the potential for CBW covert operations and terrorist incidents as adjuncts to military operations becoming more and more likely each year. With the ever-changing nature of coalition force operations coupled with the expected expansion in the spectrum of the conflict, the Canadian Forces (CF) will more likely than ever be confronted with an expansion of the battlespace, being called upon to operate beyond its traditional areas of interest. With the emergence of asymmetrical threats posed by the fragmented, worldwide proliferation of CBW technology and delivery capabilities, the threat of the use of CBW weapons against CF is likely to increase, particularly in deployments where CF are operating as part of a multi-national force required to fight in or protect urban areas, whether in battle, peacemaking, peacekeeping, or counter-terrorist operations. For an effective defence against the use of CBW agents in an urban environment, the CF will require an accurate urban dispersion model to track the agent release in the urban area, to assess the hazard, to predict the expected number of casualties and injuries and subsequent human performance degradation, and to develop the operational procedures and regulations required to control, mitigate, and monitor the fate of the CBW agent in the urban area. However, it should be emphasized that the science of turbulent diffusion in a built-up domain consisting of large groups of obstacles, which underpins these important practical problems, is still not understood to enough depth to allow (in general) well-founded, reliable, and practically useful models for urban dispersion to be constructed. While undeniable, this fact is undoubtedly surprising to most non-experts; nor, regrettably, has it inhibited many people from producing and selling software which, however attractively packaged, cannot fulfill the purposes for which it was purchased because it is based on inadequate, often wrong, science.

Proper understanding of the turbulent dispersion of contaminants to and from urban canopies requires an understanding of the atmospheric turbulence within and above the canopy. Arguably, the most important scientific challenge facing the modelling of dispersion in an urban canopy is to improve our understanding of, and means to predict, the spatial

variation of turbulent wind flow through a built-up domain. Indeed, there is an important reason for wishing to predict the nature of turbulent flow through an urban canopy on the basis of a few simple inputs (e.g., a physical description of the urban canopy and values of wind-speed and direction at some reference height above the canopy). Prediction of urban canopy flow is pre-requisite to or co-requisite with prediction of of contaminant CBW agent) dispersion within an urban canopy. Physically-based models using the Lagrangian stochastic theory of turbulent dispersion [1] are enticingly simple in concept and can be used to calculate dispersion in an urban canopy, but these approaches require that mean flow and turbulence quantities (e.g., the velocity variances, the shear stresses, the dissipation of turbulence kinetic energy, etc.) be supplied as external inputs to the model.

An urban canopy consists of a large collection of buildings and other obstacles (e.g., cars lining a street, treed areas in city green spaces, etc.) that are aggregated into complex structures. When this rough surface interacts with the environmental airflow above and within it, the flow field can become extremely complex. Curved mean streamlines, large velocity gradients, sharp velocity discontinuities, flow separations and reattachments, cavity regions, re-circulation zones, various canyon flow regimes, and strongly inhomogeneous turbulence can disperse contaminants in a highly complicated manner that is related to the particular source configuration and building geometry. Specifically, for large groups of buildings and obstacles comprising an urban canopy, the flow around any given building can be strongly affected by the average effects of all the others. The variety of turbulent flows through groups of obstacles is too large to be catalogued. In consequence, it would be naïve to expect a universal pattern in these disturbed flows, given that there are a great many length scale ratios involved in the urban flows (viz., there are the various obstacle dimensions and separations that define the topological complexity of the urban array, as well as the length scales pertaining to the spectral content of the atmospheric approach flow). Given the complexities in the atmospheric flow through groups of buildings and obstacles comprising an urban canopy, the purpose of the present study is to derive systematically a set of constitutive equations for computation of disturbed mean and turbulent flows and extraction of the requisite mean and turbulence flow quantities (e.g., mean velocity, velocity variances, shear stresses, turbulence kinetic energy, and dissipation rate) within and above an urban canopy of regular structures. The basic goal is to construct an urban flow model with a minimum amount of complexity that nevertheless captures the essence of the relevant physics of disturbed flows through a built-up domain that can be directly used to "drive"

a generalized dispersion model for the prediction of urban dispersion.

II. MODEL FOR DISTURBED MEAN VELOCITY FIELD

This section will focus, not on the mean flow around a single, isolated building, but on the spatially resolved properties of the disturbed mean flow about an extensive array of buildings with an arbitrary packing density. This building array configuration covers a broad range of problems ranging from flow around well-spaced, essentially isolated buildings at one extreme to average effects on the flow of many closely packed buildings at the other extreme. The case of greatest concern here is the intermediate case where the buildings are close enough to each other for the flows around neighboring buildings to affect each other to various degrees of interaction.

Simulations of disturbed wind flows in an urban canopy will be based on Reynolds averaging the Navier-Stokes equations so as to obtain the governing equations for the time-averaged flow statistics. To this purpose, we express the instantaneous velocity $u_i(\mathbf{x},t)$ at position $\mathbf{x}$ (with $\mathbf{x}$ referenced to a right-handed Cartesian coordinate system) and time t, as the sum of a mean $U_i(\mathbf{x},t)$, and a fluctuating part $u_i'(\mathbf{x},t)$, so that

$$u_i(\mathbf{x},t) = U_i(\mathbf{x},t) + u_i'(\mathbf{x},t). \tag{1}$$

U_i can either be an ensemble-average or time-average (in the case of a statistically stationary flow) value of u_i. By convention, the instantaneous variable is denoted by a lower-case symbol, the mean is denoted by the corresponding upper-case symbol and the fluctuating part is the lower-case symbol with a prime. Sometimes, we shall use an overbar to denote single-point ensemble (or time) averages (e.g., $U_i \equiv \overline{u}_i$). In this paper, we shall frequently use meteorological rather than tensor notation, writing $\mathbf{x} \equiv x_i = (x,y,z)$ for the position vector and $\mathbf{U} \equiv U_i = (U,V,W)$ for the mean velocity vector with the x-coordinate in the mean streamwise direction and the z-coordinate normal to the ground.

The first constraint on the disturbed mean flow within and above an urban canopy is imposed by the continuity equation. Because urban flows are small Mach number (Ma) flows (Ma << 1) (where Mach number is the ratio of the flow speed to the speed of sound in the fluid), the flow is essentially incompressible (i.e., the speed of sound c in the fluid is assumed to be essentially infinite). The continuity equation for the disturbed mean flow

reduces to an expression of the incompressibility of the fluid:

$$\frac{\partial U_i}{\partial x_i} \equiv \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0, \tag{2}$$

where repeated indices are assumed to follow the usual Einstein summation convention.

For an adiabatic flow where buoyancy forces are negligible, the Reynolds-averaged Navier-Stokes equations for mean momentum conservation at a single point assume the form

$$\frac{\partial U_i}{\partial t} = -U_j \frac{\partial U_i}{x_j} - \frac{\partial \overline{u_i' u_j'}}{\partial x_j} - 2e_{ijk}\Omega_j U_k - \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j}. \tag{3}$$

Here, P is the mean kinematic pressure, ν is the kinematic viscosity, Ω_j is the angular velocity vector of the Earth's rotation, and e_{ijk} is the Levi-Civita or permutation tensor. Furthermore, $\overline{u_i' u_j'}$ is the statistical covariance between u_i-fluctuations and u_j-fluctuations, which in physical terms is the turbulent stress. known as the Reynolds stress Equation (3) is valid in the air space of the urban canopy. In principle, the influence of the obstacles on the mean flow can be incorporated by imposing no-slip/no-leak boundary conditions over all the surfaces of the obstacles in the urban canopy. In general, this could be a complex undertaking requiring the generation of a body-fitted coordinate system for all the obstacles in the urban array.

Alternatively, it could be argued that if we are only interested in predicting the flow statistics in an urban canopy so that they can be used to "drive" a generalized dispersion model, then the resolution of the viscous boundary layers along each surface of the obstacle is not as important as predicting the large-scale re-circulation patterns in the mean flow resulting from blocking of the flow by the obstacles. In this sense, details of the flow on the characteristic scale η of the viscous boundary-layer formed on each obstacle surface are of no interest or importance. If this is so, it would be much simpler to deal with the effect of the obstacles on the flow by parameterizing obstacle-drag by means of a bulk momentum sink (rather than imposing no-slip/no-leak boundary conditions on a complex surface), implying that we can forsake details of the flow on the scale η What is important for proper calculation of the flow in the urban canopy is that the correct amount of momentum should be removed *near* each obstacle. Here, we wish to resolve *important* flow gradients—important presumably meaning, significant on scales *much larger than* η. Hence, the present work is implicitly based on the following premise: namely, that an array of obstacles constituting an urban canopy generates mean flow patterns that are resolvable by the simple flow model

to be constructed presently, even though that model is presumably incapable of capturing the nuances of the flow on each individual wall, and that the scale and intensity of the mean flow zonal patterns might be indicative of the intermittent large-scale eddies.

To this purpose, it is noted that the integration of the pressure over the element surfaces of an obstacle within an averaging volume V is the exact expression for the momentum sink consisting of the pressure (form) drag created by the obstacle element. Hence, the pressure or form drag has the exact form

$$f_{F_i} = \frac{1}{V} \iint_{S_I} P n_i \, dS, \tag{4}$$

where S_I denotes the sum of all the obstacle surfaces in V; and, n_i is the unit normal vector on S_I, positive into the air. The form drag of Equation (4) represents the force exerted by a canopy element (obstacle) on a unit mass of air within V. This drag force term may be modelled using the conventional expression [2]

$$f_{F_i} = -C_d A(x, y, z) U_i \|\mathbf{U}\|, \tag{5}$$

where C_d is the drag coefficient assigned to a *single* canopy element (obstacle), A is the obstacle frontal area per unit volume of space, and $\|\cdot\|$ denotes the Euclidean norm so that $\|\mathbf{U}\| \equiv \left(U^2 + V^2 + W^2\right)^{1/2}$. Note that this parameterization for the drag force explicitly incorporates the fact that the drag always acts against the direction of the flow. It should be emphasized that the representation of an obstacle in terms of a drag coefficient is an heuristic approach and implicitly assumes that the obstacle is porous (leaky). However, as $C_d A$ becomes large, the obstacle becomes decreasingly permeable, and perhaps one can use this representation to attain the simulation of a bluff body flow without the complication of requiring a dense and complex (body-fitted) grid. This may be a viable representation for a single obstacle. Another interesting aspect of this overall approach is that one such porous element ($C_d A$ non-zero, but not as large as the value for a single obstacle) might be interpreted to represent a whole group of obstacles. Furthermore, in the proposed model, there is not the slightest difficulty to include treed areas (e.g., parks) interspersed between the building elements. Hence, considerable local heterogeneity (such as is evident in any city) could be represented in the model.

Including the drag force in Equation 2 and assuming a statistically stationary flow, the mean momentum equations expressed in meteorological notation now assume the fol-

lowing forms:

$$\frac{\partial}{\partial x}(U^2+\overline{u'^2})+\frac{\partial}{\partial y}(UV+\overline{u'v'})+\frac{\partial}{\partial z}(UW+\overline{u'w'})=f(V-V_G)-\frac{\partial P}{\partial x} - C_dAU\sqrt{U^2+V^2+W^2}, \quad (6)$$

$$\frac{\partial}{\partial x}(UV+\overline{u'v'})+\frac{\partial}{\partial y}(V^2+\overline{v'^2})+\frac{\partial}{\partial z}(VW+\overline{v'w'})=f(U_G-U)-\frac{\partial P}{\partial y} - C_dAV\sqrt{U^2+V^2+W^2}, \quad (7)$$

and

$$\frac{\partial}{\partial x}(UW+\overline{u'w'})+\frac{\partial}{\partial y}(VW+\overline{v'w'})+\frac{\partial}{\partial z}(W^2+\overline{w'^2})=-\frac{\partial P}{\partial z} - C_dAW\sqrt{U^2+V^2+W^2}, \quad (8)$$

corresponding to the streamwise, cross-stream, and vertical mean momentum balance, respectively. In Equations (6) and (7), f is the Coriolis parameter defined as $f = 2\omega\sin\phi$, where ω is the angular velocity of the Earth's rotation, and ϕ is latitude. Furthermore, the large-scale background horizontal kinematic pressure gradient P_b in the Planetary Boundary-Layer (PBL) has been parameterized in terms of the geostrophic wind, so that $fU_G = -\partial P_b/\partial y$ and $fV_G = \partial P_b/\partial x$, where (U_G, V_G) are the components of a nominal geostrophic wind aloft. We have written Equations (6) to (8) in conservation form (i.e., the divergence term involving the momentum flux appears on the left-hand side) in order to facilitate the construction of a numerical solution. Furthermore, the molecular transport of momentum has been neglected compared to the turbulent transport.

To proceed, we need a prescription for computing the Reynolds stresses $\overline{u_i'u_j'}$ in the mean momentum equations. Our intent here is to uncover a simple but workable closure for the disturbed urban canopy flow. Towards this objective, we note that in most disturbed thin shear-layer flows (i.e., flows wherein shear strains $\gg$ normal strains) normal stress gradients play a much smaller role than the shear stress gradients. Their inclusion via some closure assumption does have the purely computational advantage of providing diffusion terms. For simplicity, we will neglect the normal stress gradients in Equations (6) to (8) The remaining shear stress terms will be modelled using K-theory (Boussinesq eddy-viscosity approximation), in which the specific Reynolds-stress tensor components are modelled for arbitrary three-dimensional flows as [2]

$$\overline{u_i'u_j'} = -K\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)+\frac{2}{3}k\delta_{ij}, \qquad i,j=1,2,3, \quad (9)$$

where k is the turbulence kinetic energy (TKE) and δ_{ij} is the Kronecker delta function. With these assumptions, the mean momentum equations simplify to the following forms:

$$\frac{\partial}{\partial x}\left(U^2 - K_a\frac{\partial U}{\partial x}\right) + \frac{\partial}{\partial y}\left(UV - K\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)\right) + \frac{\partial}{\partial z}\left(UW - K\left(\frac{\partial U}{\partial z} + \frac{\partial W}{\partial x}\right)\right) = f(V - V_G) - \frac{\partial P}{\partial x} - C_d AU\sqrt{U^2+V^2+W^2}, \quad (10)$$

$$\frac{\partial}{\partial x}\left(UV - K\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)\right) + \frac{\partial}{\partial y}\left(V^2 - K_a\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(VW - K\left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y}\right)\right) = f(U_G - U) - \frac{\partial P}{\partial y} - C_d AV\sqrt{U^2+V^2+W^2}, \quad (11)$$

and

$$\frac{\partial}{\partial x}\left(UW - K\left(\frac{\partial U}{\partial z} + \frac{\partial W}{\partial x}\right)\right) + \frac{\partial}{\partial y}\left(VW - K\left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y}\right)\right) + \frac{\partial}{\partial z}\left(W^2 - K_a\frac{\partial W}{\partial z}\right) = -\frac{\partial P}{\partial z} - C_d AW\sqrt{U^2+V^2+W^2}. \quad (12)$$

In Equations (10) to (12), we added diffusion terms where diffusion was otherwise absent. We regard these as artificial, and signify artificial diffusivity as K_a, which is very small.

We chose to specify the eddy viscosity as:

$$K(x,y,z) = \lambda(x,y,z)\sqrt{c_e k(x,y,z)}, \quad (13)$$

where λ is the turbulence length scale. Note that $k^{1/2}$ has been used in this specification as the turbulence velocity scale. The constant, c_e, is the equilibrium shear-stress/TKE ratio (u_{*0}^2/k_0) immediately above the canopy, which if evaluated in the local equilibrium region varies only slightly from one wall-shear layer to another. The TKE, k, required to specify K will be obtained from a simplified transport equation; and, the turbulence length scale, λ, will be specified algebraically.

In specifying the turbulence length scale, we wish to reproduce the observed characteristic [3] of a constant length scale across much of the upper canopy ($\lambda < k_v h_c$, where $k_v \approx 0.4$ is von Kármán's constant and h_c is canopy height), and some distance above but near the canopy an 'inertial sub-layer' wherein $\lambda \propto (z-d)$, d being the displacement height. Furthermore, we wish also to allow the length scale to (realistically) vanish on the ground,

i.e. $\lambda \to 0$ as $z/h_c \to 0$. In view of these requirements, we model the turbulence length scale as $\lambda = \max(\lambda_i, \lambda_o)$ where:

$$\frac{1}{\lambda_i} = \frac{1}{k_v z} + \frac{1}{\lambda_c} \qquad \text{and} \qquad \frac{1}{\lambda_o} = \frac{1}{k_v (z-d)} + \frac{1}{L_\infty}. \tag{14a}$$

The 'outer' scale λ_o recognizes the displacement (of the equilibrium structure) due to the canopy and, through the imposed (i.e., maximum permitted) value L_∞, the fact of a restricted depth to the planetary boundary-layer (PBL). In the case of a full PBL simulation, L_∞ should limit the length scale to the PBL depth which is often estimated [4] as approximately

$$L_\infty = \frac{0.0004 U_G}{f}. \tag{14b}$$

The 'inner' scale λ_i recognizes both the limitation on eddy size by proximity to the ground and the presence of the canopy. Raupach *et al.* [5] have suggested that the strength of the wind shear at canopy height is critical to eddy transport within a canopy, defining a shear length scale, λ_c [cf. Equation (14a)]:

$$\lambda_c(x,y) = \frac{c k^{1/2}(x,y,z=h_c)}{\left.\dfrac{\partial S}{\partial z}\right|_{z=h_c}}, \tag{15}$$

where $S \equiv (U^2 + V^2)^{1/2}$ is the horizontal windspeed and c is an order one (viz., $c \sim O(1)$) 'closure constant'. The shear length scale has been shown [5] to play a critical role in the dynamical process that produces the dominant canopy eddies.

In Equation (14a), the displacement length d is required to specify the outer length scale λ_o. In this model, we use the center-of-pressure theorem [6] to determine the displacement height for the urban canopy. This theorem states that d can be regarded as the level at which the mean drag on the surface appears to act, so that

$$d = \frac{\displaystyle\int_0^{h_c} z \langle U \rangle^2 \, dz}{\displaystyle\int_0^{h_c} \langle U \rangle^2 \, dz}. \tag{16}$$

In Equation (16), the angle brackets are used to denote a spatial average. Because neither U or its spatial average $\langle U \rangle$ are known *a priori*, d needs to be calculated iteratively at the end of each outer iteration using Equation (16) (bootstrap procedure). The numerical solution of the partial differential equations to determine the mean momentum will be discussed later in this report.

The turbulence kinetic energy, k, required in the specification of K [cf. Equation (13)] is determined from the approximate TKE transport equation:

$$\frac{\partial}{\partial x}\left(Uk - \mu K\frac{\partial k}{\partial x}\right) + \frac{\partial}{\partial y}\left(Vk - \mu K\frac{\partial k}{\partial y}\right) + \frac{\partial}{\partial z}\left(Wk - \mu K\frac{\partial W}{\partial z}\right)$$
$$= K\left[\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial z} + \frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y}\right)^2\right]$$
$$+ 2K\left[\left(\frac{\partial U}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial W}{\partial z}\right)^2\right] + P_w - \epsilon, \quad (17)$$

where P_w is the wake production term, ϵ is the viscous dissipation of TKE, and the constant μ represents the ratio of the effective eddy diffusivity for TKE to the eddy viscosity (i.e., inverse of a turbulent Schmidt number). The wake production term arises from the creation of TKE as the mean flow does work against the canopy drag (viz., it represents the conversion of mean kinetic energy (MKE) into turbulence in the wakes of the obstacles). It is formally equal to the rate of working of the mean flow against the pressure or form drag of the obstacles, so that

$$P_w = -\frac{1}{2}U_i f_{F_i} = \frac{1}{2}C_d A\|\mathbf{U}\|^3. \quad (18)$$

The TKE dissipation, ϵ, is modelled as $\epsilon = \max(\epsilon_{cc}, \epsilon_{fd})$ where:

$$\epsilon_{cc} = \frac{(c_\epsilon k)^{3/2}}{\lambda}, \qquad \epsilon_{fd} = \alpha_* C_d A k\sqrt{U^2 + V^2 + W^2}. \quad (19)$$

The term ϵ_{cc} is the standard parameterization for viscous dissipation, the only TKE sink above the canopy (ϵ_{cc} balances shear production of TKE, in the local equilibrium layer well above the canopy). Our specification for ϵ_{fd} ('form drag'), which converts resolved TKE to the small, rapidly dissipated 'wake-scales', can be justified on physical grounds. We expect the closure coefficient α_* to be $O(1)$, but its exact value depends on the partitioning of k among its components (u', v', w').

III. PARTITIONING OF THE TURBULENCE KINETIC ENERGY

Generalized dispersion models (such as a Lagrangian stochastic model) for predicting the transport and dispersion of contaminants in the atmosphere require the turbulent normal stresses $\overline{u'^2}$, $\overline{v'^2}$, and $\overline{w'^2}$. The estimates of these quantities, therefore, will significantly affect the performance of the dispersion models. Hence, the problem of the 'correct' partitioning

of the TKE, $k \equiv \frac{1}{2}(\sigma_u^2 + \sigma_v^2 + \sigma_w^2)$, into the normal stress components $\sigma_u^2 \equiv \overline{u'^2}$, $\sigma_v^2 \equiv \overline{v'^2}$, and $\sigma_w^2 \equiv \overline{w'^2}$ is critically important. The simplest partitioning of the TKE would be simply to assume that an isotropic partitioning holds within and above the canopy, so that $\sigma_u^2 = \sigma_v^2 = \sigma_w^2 = \frac{2}{3}k$. However, this is usually not consistent with the observed equilibrium partitioning of the TKE, where typically $\sigma_{u0}/u_{*0} \approx \sigma_{v0}/u_{*0} \approx 2.0$ and $\sigma_{w0}/u_{*0} \approx 1.2$ in the logarithmic, constant-stress layer above the canopy. In view of this, an equilibrium partitioning of the TKE can be assumed to hold throughout the canopy (viz., the ratio of normal stresses in the canopy is assumed to be equal to that in the equilibrium layer above the canopy).

The isotropic or equilibrium partitioning of the TKE is rather arbitrary, and no experimental evidence exists to support this particular form of partitioning. In view of this deficiency, we will construct an algebraic Reynolds stress model (ASM) that can be used to partition the TKE. To this end, we will make use of a well understood and successful existing second-order closure model for the Reynolds stresses, namely that of Launder *et al.* [7], which we have modified in order to include the extra physics relating to canopy flow. The extra physics that an urban canopy flow model must reproduce devolves simply from the interaction of the airflow and the obstacles. We confine all modifications to additional terms required to describe the physics peculiar to canopy flow so that the model relaxes to the well tested free-air behavior as the obstacles disappear.

The budget equations for the shear and normal stresses may be written symbolically as

$$\frac{D\overline{u_i'u_j'}}{Dt} = P_{ij}^s + P_{ij}^w + \Pi_{ij} + PT_{ij} - \epsilon_{ij}, \tag{20}$$

where: P_{ij}^s is the shear production term, P_{ij}^w is the wake production term, Π_{ij} is the pressure-strain term (or pressure redistribution term), PT_{ij} is the turbulent and pressure transport term, and ϵ_{ij} is the dissipation term. Here, D/Dt is the substantial (material) derivative (i.e., the time rate of change following a moving fluid element). All the terms in Equation (20) are conventional, with the exception of P_{ij}^w and the modification to ϵ_{ij} required, respectively, to represent the working of the mean flow against the canopy drag and the additional dissipation arising from the interaction of the flow with the obstacles. Noting that $k \equiv \frac{1}{2}\overline{u_i'u_i'}$, the transport equation for k can be obtained from Equation (20) by contracting the indices i and j and dividing the result by two to give:

$$\frac{Dk}{Dt} = P_s + P_w + PT - \epsilon, \tag{21}$$

where $P_s \equiv \frac{1}{2}P^s_{ii}$ is the mechanical or shear production term for TKE, $P_w \equiv \frac{1}{2}P^w_{ij}$ is the wake production term for TKE, $PT \equiv \frac{1}{2}PT_{ii}$ is the turbulent and pressure transport of TKE, and $\epsilon = \frac{1}{2}\epsilon_{ii}$ represents the viscous dissipation of TKE. The pressure redistribution term Π_{ij}, which expresses the rate at which energy is distributed between the normal stresses, does not contribute to the TKE budget equation because $\Pi_{ii} = 0$.

In order to derive an algebraic stress model from Equations (20) and (21) that can be used for partitioning the TKE, we adopt a closure approximation proposed by Rodi [8]. Rodi assumed that the overall transport of $\overline{u_i'u_j'}$, including the local rate of change, advection, and turbulent and pressure transport terms, is proportional to the overall transport of the TKE k, with the proportional factor (which is not a constant) being the ratio $\overline{u_i'u_j'}/k$. This assumption gives

$$\frac{D\overline{u_i'u_j'}}{Dt} - PT_{ij} = \frac{\overline{u_i'u_j'}}{k}\left(\frac{Dk}{Dt} - PT\right). \tag{22a}$$

Substituting Equations (20) and (21) into Equation (22a) gives

$$\frac{\overline{u_i'u_j'}}{k}(P_s + P_w - \epsilon) = P^s_{ij} + P^w_{ij} + \Pi_{ij} - \frac{2}{3}\epsilon\delta_{ij}. \tag{22b}$$

In Equation (22b), we have implicitly assumed that ϵ_{ij} vanishes unless $i = j$. Hence, we have modelled the dissipation term using the conventional isotropic form $\epsilon_{ij} = \frac{2}{3}\epsilon\delta_{ij}$. The shear production term is determined exactly as

$$P^s_{ij} \equiv -\left(\overline{u_i'u_k'}\frac{\partial U_j}{\partial x_k} + \overline{u_j'u_k'}\frac{\partial U_i}{\partial x_k}\right). \tag{23}$$

The wake production term will be modelled as

$$P^w_{ij} = C_d A\|\mathbf{U}\|^3\frac{\delta_{ij}}{3}. \tag{24}$$

The key term in the partitioning of the TKE is Π_{ij} (pressure-strain term) which is modelled after Launder *et al.* [7] as

$$\begin{aligned}\Pi_{ij} = &-c_1\frac{\epsilon}{k}\left(\overline{u_i'u_j'} - \frac{2}{3}\delta_{ij}k\right) - \frac{c_2+8}{11}\left(P^s_{ij} - \frac{2}{3}P_s\delta_{ij}\right)\\ &- \frac{30c_2-2}{55}k\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{8c_2-2}{11}\left(D_{ij} - \frac{2}{3}P_s\delta_{ij}\right),\end{aligned} \tag{25}$$

where

$$D_{ij} \equiv -\left(\overline{u_i'u_k'}\frac{\partial U_k}{\partial x_j} + \overline{u_j'u_k'}\frac{\partial U_k}{\partial x_i}\right), \tag{26}$$

and c_1 and c_2 are closure coefficients.

If we substitute Equations (23) to (26) (and their contracted forms) into Equation (22b), it can be shown (after some lengthy and tedious algebraic manipulations) that

$$\frac{a_{ij}}{g^*} + b_2\left(S_{ik}a_{kj} + S_{jk}a_{ki} - \frac{2}{3}S_{lk}a_{kl}\delta_{ij}\right) + b_3\left(W_{ik}a_{kj} + W_{jk}a_{ki}\right) = -b_1 S_{ij}, \tag{27}$$

where

$$a_{ij} \equiv \frac{\overline{u_i' u_j'}}{k} - \frac{2}{3}\delta_{ij} \tag{28}$$

is the anisotropy stress tensor;

$$S_{ij} \equiv \frac{1}{2}\frac{k}{\epsilon}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{29}$$

is the normalized mean rate-of-strain tensor (viz., normalized by the turbulence time scale k/ϵ);

$$W_{ij} \equiv \frac{1}{2}\frac{k}{\epsilon}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right) \tag{30}$$

is the normalized mean vorticity (rotation) tensor;

$$g^* \equiv \frac{1}{c_1 + P_s/\epsilon + P_w/\epsilon - 1}; \tag{31}$$

and,

$$b_1 \equiv \frac{8}{15}, \quad b_2 \equiv \frac{5 - 9c_2}{11}, \quad b_3 \equiv \frac{7c_2 + 1}{11}, \tag{32}$$

are numerical constants. Note that g^* [cf. Equation (31)] embodies the information on the net production (shear and wake) per unit dissipation of the TKE. Equation (27) constitutes a closed system of algebraic equations for the determination of the Reynolds stress anisotropy in terms of the mean velocity gradients. Once the mean flow and TKE and its dissipation are known (i.e., predicted using the urban flow model described in the previous section), Equation (27) can then be solved numerically at each point in the flow domain for the anisotropy stress components (which, in turn, determines the partitioning of the TKE).

It is interesting to note since Π_{ij} in Equation (25) is tensorially linear in the anisotropy stress tensor, it should be possible to obtain an explicit expression for a_{ij} in terms of the mean velocity gradients. Pope [9] showed that this can be accomplished for two-dimensional turbulent flows by using techniques from linear algebra that make use of integrity bases (viz., polynomial representations for isotropic tensor functions). This was

extended to three-dimensional turbulent flows by Gatski and Speziale [10], who overcame the severe complexity of the algebra involved in the construction of an explicit solution by using a symbolic manipulation program (namely, *Mathematica* [11]). Although both Pope's and Gatski and Speziale's papers seem to have gone unnoticed to some extent, they contain nice features that drew our attention. Specifically, the formulation seems promising in providing an efficient and adequate tool for the study of Reynolds stresses in a complex built-up domain.

To apply Gatski and Speziale's [10] formulation in constructing an explicit solution to Equation (27), it is convenient to introduce the dimensionless, rescaled variables:

$$\tilde{S}_{ij} \equiv g^* b_2 S_{ij}, \tag{33}$$

$$\tilde{a}_{ij} \equiv \frac{b_2}{b_1} a_{ij}, \tag{34}$$

and

$$\widetilde{W}_{ij} \equiv g^* \frac{b_3}{b_1} W_{ij}. \tag{35}$$

Introduction of these rescaled variables into Equation (27) then yields the canonical form:

$$\tilde{a}_{ij} = \tilde{S}_{ij} - \left(\tilde{S}_{ik}\tilde{a}_{kj} + \tilde{S}_{jk}\tilde{a}_{ki} - \frac{2}{3}\tilde{S}_{lk}\tilde{a}_{kl}\delta_{ij} \right) - \left(\widetilde{W}_{ik}\tilde{a}_{kj} + \widetilde{W}_{jk}\tilde{a}_{ki} \right). \tag{36}$$

Starting from Equation (36), Gatski and Speziale's explicit solution for $\tilde{a}_{ij}$ in terms of $\tilde{S}_{ij}$ and $\widetilde{W}_{ij}$ may be expressed as follows:

$$\tilde{a}_{ij} = \sum_{\lambda} G^{(\lambda)} T_{ij}^{(\lambda)}, \tag{37}$$

where $T_{ij}^{(\lambda)}$ is the integrity basis for functions of a symmetric ($\tilde{S}_{ij}$) and antisymmetric ($\widetilde{W}_{ij}$) tensor and $G^{(\lambda)}$ are scalar functions of the irreducible invariants of $\tilde{S}_{ij}$ and $\widetilde{W}_{ij}$.

In three-dimensional space, the integrity basis constructed from the symmetric $\tilde{S}_{ij}$ and antisymmetric $\widetilde{W}_{ij}$ tensors is given by (e.g., Spencer [12])

$$T_{ij}^{(1)} = \tilde{S}_{ij}, \tag{38a}$$

$$T_{ij}^{(2)} = \tilde{S}_{ik}\widetilde{W}_{kj} - \widetilde{W}_{ik}\tilde{S}_{kj}; \tag{38b}$$

$$T_{ij}^{(3)} = \tilde{S}_{ik}\tilde{S}_{kj} - \frac{1}{3}\mathrm{tr}[\tilde{\mathbf{S}}^2]\delta_{ij}; \tag{38c}$$

$$T_{ij}^{(4)} = \widetilde{W}_{ik}\widetilde{W}_{kj} - \frac{1}{3}\mathrm{tr}[\widetilde{\mathbf{W}}^2]\delta_{ij}; \tag{38d}$$

$$T_{ij}^{(5)} = \widetilde{W}_{ik}\widetilde{S}_{kl}\widetilde{S}_{lj} - \widetilde{S}_{ik}\widetilde{S}_{kl}\widetilde{W}_{lj}; \tag{38e}$$

$$T_{ij}^{(6)} = \widetilde{W}_{ik}\widetilde{W}_{kl}\widetilde{S}_{lj} + \widetilde{S}_{ik}\widetilde{W}_{kl}\widetilde{W}_{lj} - \frac{2}{3}\mathrm{tr}[\widetilde{\mathbf{S}}\widetilde{\mathbf{W}}^2]\delta_{ij}; \tag{38f}$$

$$T_{ij}^{(7)} = \widetilde{W}_{ik}\widetilde{S}_{kl}\widetilde{W}_{lm}\widetilde{W}_{mj} - \widetilde{W}_{ik}\widetilde{W}_{kl}\widetilde{S}_{lm}\widetilde{W}_{mj}; \tag{38g}$$

$$T_{ij}^{(8)} = \widetilde{S}_{ik}\widetilde{W}_{kl}\widetilde{S}_{lm}\widetilde{S}_{mj} - \widetilde{S}_{ik}\widetilde{S}_{kl}\widetilde{W}_{lm}\widetilde{S}_{mj}; \tag{38h}$$

$$T_{ij}^{(9)} = \widetilde{W}_{ik}\widetilde{W}_{kl}\widetilde{S}_{lm}\widetilde{S}_{mj} - \frac{2}{3}\mathrm{tr}[\widetilde{\mathbf{S}}^2\widetilde{\mathbf{W}}^2]\delta_{ij}; \tag{38i}$$

$$T_{ij}^{(10)} = \widetilde{W}_{ik}\widetilde{S}_{kl}\widetilde{S}_{lm}\widetilde{W}_{mn}\widetilde{W}_{nj} - \widetilde{W}_{jk}\widetilde{W}_{kl}\widetilde{S}_{lm}\widetilde{S}_{mn}\widetilde{W}_{nj}. \tag{38j}$$

Here, $\mathbf{S} \equiv (S_{ij})$, $\mathbf{W} \equiv (W_{ij})$, etc. and $\mathrm{tr}(\cdot)$ denote the trace operation. The expansion coefficients $G^{(\lambda)}$ in Equation (37) are scalar functions of the irreducible invariants of $\widetilde{S}_{ij}$ and $\widetilde{W}_{ij}$. The irreducible invariants of $\widetilde{S}_{ij}$ and $\widetilde{W}_{ij}$ are:

$$\xi_1 = \mathrm{tr}[\widetilde{\mathbf{S}}^2]; \tag{39a}$$

$$\xi_2 = \mathrm{tr}[\widetilde{\mathbf{W}}^2]; \tag{39b}$$

$$\xi_3 = \mathrm{tr}[\widetilde{\mathbf{S}}^3]; \tag{39c}$$

$$\xi_4 = \mathrm{tr}[\widetilde{\mathbf{S}}\widetilde{\mathbf{W}}^2]; \tag{39d}$$

$$\xi_5 = \mathrm{tr}[\widetilde{\mathbf{S}}^2\widetilde{\mathbf{W}}^2]. \tag{39e}$$

Gatski and Speziale's [10] explicit results for the expansion coefficients $G^{(\lambda)}$ can be expressed as

$$G^{(1)} = -\frac{1}{2}\left(6 - 3\xi_1 - 21\xi_2 - 2\xi_3 + 30\xi_4\right)/D; \tag{40a}$$

$$G^{(2)} = -\left(3 + 3\xi_1 - 6\xi_2 + 2\xi_3 + 6\xi_4\right)/D; \tag{40b}$$

$$G^{(3)} = \left(6 - 3\xi_1 - 12\xi_2 - 2\xi_3 - 6\xi_4\right)/D; \tag{40c}$$

$$G^{(4)} = -3\left(3\xi_1 + 2\xi_3 + 6\xi_4\right)/D; \tag{40d}$$

$$G^{(5)} = -9/D; \tag{40e}$$

$$G^{(6)} = -9/D; \tag{40f}$$

$$G^{(7)} = 9/D; \tag{40g}$$

$$G^{(8)} = 9/D; \tag{40h}$$

$$G^{(9)} = 18/D; \tag{40i}$$

$$G^{(10)} = 0; \tag{40j}$$

where the denominator D is given by

$$D = 3 - \frac{7}{2}\xi_1 + \xi_1^2 - \frac{15}{2}\xi_2 - 8\xi_1\xi_2 + 3\xi_2^2 - \xi_3 + \frac{2}{3}\xi_1\xi_3 - 2\xi_2\xi_3 + 21\xi_4 + 24\xi_5 + 2\xi_1\xi_4 - 6\xi_2\xi_4. \tag{41}$$

In summary, Equations (37), (38), (39), (40), and (41) provide an explicit expression for the partitioning of the TKE. whereby the normal stresses can be expressed explicitly in terms of the mean velocity gradients.

IV. NUMERICAL DETAILS AND BOUNDARY CONDITIONS

The four partial differential equations governing the mean velocity $\mathbf{U} \equiv (U, V, W)$ [cf. Equations (10) to (12)] and the TKE k [cf. Equation (17)] can be expressed in the following generic form:

$$\frac{\partial}{\partial x_j}\left[U_j\phi - \Gamma\frac{\partial\phi}{\partial x_j}\right] = S_C + S_P\phi, \qquad (42)$$

where ϕ is taken to represent one of the four dependent variables, namely U, V, W, or k. In this form, these four equations can be linearized and decoupled by assuming that ϕ, Γ, S_C and S_P are functions of position only and have values calculated from the previous iteration cycle. The equations are then discretized by use of the finite volume technique in which the partial differential equations are integrated over appropriate finite volumes on a staggered grid to obtain difference equations. In performing this integration, the power-law scheme recommended by Patankar [13] was used to discretize the convection and diffusion terms in the generic form of the conservation equation exhibited in Equation (42). The pressure-correction equation is calculated using a variant of Patankar's [13] well-documented Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) known as the SIMPLEC (SIMPLE Consistent) method [14], with the difference that the momentum equations are manipulated so that the SIMPLEC velocity correction equations omit terms that are less significant that those omitted in SIMPLE.

In the SIMPLE method (or, its variant SIMPLEC), the computational domain is discretized into finite control volumes and the governing equations (mean momentum and TKE transport equations) are integrated within their respective control volumes to obtain difference equations which state that flux differences across the faces of the control volume must balance the change in storage due to source/sinks in the control volume. Thus,

$$J_E - J_W + J_N - J_S + J_T - J_B = SU\delta x\delta y\delta z, \qquad (43)$$

where J is the (advective plus diffusive) flux of ϕ [cf. Equation (42)] across control volume faces (labelled E, W, N, S, T, B for east, west, north, south, top, bottom faces, respectively). For example, one component of the source/sink term in the U-momentum equation

[cf. Equation (10)] is the form drag, which for a finite control volume will contribute the sink term $C_d A U \sqrt{U^2 + V^2 + W^2}\, \delta\Omega$, where $\delta\Omega$ is the volume of the U-control volume. The pressure term in the momentum equations is adjusted so that the continuity equation is satisfied. In particular, the continuity equation is integrated throughout the control volume, multiplied by the grid point value of the variable in question (say, U), and subtracted from the conservation equation. One finally obtains a linearized discretization equation of the form

$$a_c U_c = a_e U_e + a_w U_w + a_n U_n + a_s U_s + a_t U_t + a_b U_b + b + \text{pressure term}, \qquad (44)$$

where the neighbor coefficients (a_c, a_e, etc.) depend on the current best estimates of the unknown fields. Patankar [12] gives a set of simple rules which ensures accurate conservation and numerical stability.

The U-momentum, V-momentum, W-momentum, TKE transport, and continuity equations are solved on a staggered grid of points in space. The control volumes were here set by first specifying the locations of the U, V, and W points (through which pass the east/west, north/south, and top/bottom control volume faces, respectively, for the pressure correction equation) and then placing the pressure points at the center of the control volume thus defined. In other words, the pressure is evaluated at "ordinary" nodal points but the mean velocity components are calculated on staggered grids centered around the cell faces with the U-velocities stored at the pressure cell faces 'e' (east) and 'w' (west), the V-velocities at faces 'n' (north) and 's' (south), and the W-velocities at cell faces 't' (top) and 'b' (bottom). The staggering of the velocity avoids the unrealistic behavior of the discretised momentum equations for spatially oscillating pressures like the 'checker-board' field. A further advantage of the staggered grid arrangement is that it generates velocities at exactly the locations where they are required for the pressure-correction equation. Hence, no interpolation is needed to calculate the velocities at the pressure control volume cell faces (which, in turn, is the result of the discretized continuity equation). The grid points for the TKE k are sited at the same level as the grid points for W, but offset by half the control volume length in the x-direction so that the TKE grid points lie directly above those for U. This permits the principal contributions (e.g., $K(\partial U/\partial z)^2$) to the shear production term in the transport equation for the TKE to be calculated directly without interpolation.

One cycle (outer iteration) of the computation consists of the following calculations: discretize and solve the equations for the three components of the mean velocity (U, V, W);

solve the equation for the error in the pressure δP and correct the pressures and mean velocities; discretize and solve the transport equation for the TKE k; and, use this information to update the the turbulence length scale $\lambda(x, y, z)$ and the eddy visocity $K(x, y, z)$ and begin another cycle of calculations. The resulting algebraic equations obtained from the discretization of the mean momentum and TKE equations were solved by a 3-D version of the Alternating Direction Implicit (ADI) procedure in which three sweeps of the computational domain (one in each of the coordinate directions) are done in each (inner) iteration. The tri-diagonal matrix algorithm (TDMA) was used for rapidly solving the tri-diagonal systems that result from a sweep in a given coordinate direction.

Spatially-invariant fields $U(z)$, $V(z)$, and $k(z)$, which were obtained as equilibrium ($\partial/\partial x = \partial/\partial y = 0$) solutions of the flow equations (with a uniform urban canopy layer), were used as the initial guesses for the first outer iteration or cycle. Initially, the drag term in Equations (10) to (12) and in Equation (17) was uniform across the urban layer ($z < h_c$, where h_c is the canopy height), and the inter-building open spaces (streets) were progressively, but slowly, carved out cycle after cycle. In other words, the product of the drag coefficient and frontal area density $C_d A$ was under-relaxed within the inter-building open spaces or streets via

$$C_d A = \alpha (C_d A)_{\text{target}} + (1 - \alpha)(C_d A)_{\text{old}}, \tag{45}$$

where α is the relaxation factor, which for under-relaxation requires α to be positive and less than unity. In Equation (45), the "drag" parameter $C_d A$ is slowly adjusted, during the course of the outer iterations, to ultimately achieve the desired or target value (which for open spaces is $(C_d A)_{\text{target}} = 0$). This gradual relaxation of the "drag" parameter avoids the introduction of a state of extreme imbalance in the momentum budgets by allowing the drag term to relax slowly to its final value in the inter-building open spaces, starting from the initial condition of a uniform canopy below $z = h_c$. This relaxation of the "drag" parameter merely affects the rate of progress towards the solution, and not the ultimate solution itself.

For an extensive urban array consisting of a large group of obstacles, the flow is assumed to be periodic in the lateral directions with periods L_x and L_y in the x- and y-directions, respectively. Hence, there are, in fact, no lateral boundary conditions imposed on the solution. Now, assume that the dependent variables ϕ (e.g., U, V, W, and k) are defined at locations on the x-, y-, and z-axes with the indices numbered, respectively,

as $i = 1, 2, \ldots, I-1, I, I+1, \ldots, I_{\max}$, $j = 1, 2, \ldots, J-1, J, J+1, \ldots, J_{\max}$, and $k = 1, 2, \ldots, K-1, K, K+1, \ldots, K_{\max}$. To impose lateral periodicity, two actions are necessary. Firstly, for control volumes spanning or touching the periodic boundaries in either the x- and/or y-directions, one of the neighboring values of ϕ is outside the computational domain, but is available (i.e., resolved and known) at the other extreme of the domain. For example, when $i = 1$, one will require the neighboring value $\phi(i-1, j, k)$ in the x-direction, and this is simply equal to $\phi(I_{\max}, j, k)$ by lateral periodicity. Secondly, the structure of the matrix relation in the sweeps in the x- and y-directions are not quite tri-diagonal; for following the example cited above, if $a_W(1, j, k)$ is the coefficient of the (missing, or unseen, $i - 1 = 1 - 1 = 0$) neighboring value $\phi(i-1, j, k)$ in the discretized equation for ϕ at node $(1, j, k)$, this value must be inserted in the coefficient matrix so as to multiply the appropriate substitute, namely $\phi(I_{\max}, j, k)$. Hence, one never imposes lateral periodicity by any direct action of the form $\phi(L_x, y, z) = \phi(0, y, z)$. This lack of any explicit boundary condition in the lateral directions is both elegant and natural.

The upper boundary conditions were imposed as follows. The boundary conditions imposed here depend on the extent of the flow domain modelled. If the flow covers only one or a few individual buildings (say, $L_x \approx L_y \approx$ 20-100 m), only the atmospheric surface layer [constant stress layer (CSL)] needs to be resolved. The Coriolis force can be neglected (i.e., f is set equal to zero), and far aloft the spatially-averaged mean wind direction, $\langle\beta\rangle$, is height-independent For this case, the shear stress u_* along the top boundary at $z = z_T$ is specified and serves as the upper boundary condition for the U-momentum equation. However, to investigate full-scale flows about large groups of buildings, which may require $L_x \approx L_y$ be much larger (say, $\approx$ 1000 m, or more), a full planetary boundary-layer (PBL) with Coriolis effects (and, the concomitant wind direction changes with height) needs to be included. For this case, the geostrophic wind components (U_G, V_G) were specified at the top of the PBL and serve as the upper boundary condition for the horizontal mean momentum equations. For both the CSL and PBL cases, along the top boundary at $z = z_T$, $W = 0$ and $\partial k/\partial z = 0$ were imposed.

In the staggered grid used, we set the lowest vertical velocity and TKE grid points on the ground ($z = 0$). Consequently, the lowest U, V grid points lay above the ground, at $z = z_P$, and a condition on the corresponding vertical momentum fluxes to ground was

required. We specified $\overline{u'w'} = -u_{*u}^2$, $\overline{v'w'} = -u_{*v}^2$, where

$$u_{*u} = k_v U_P / \ln\left(\frac{z_P}{z_{S0}}\right), \qquad u_{*v} = k_v V_P / \ln\left(\frac{z_P}{z_{S0}}\right), \tag{46a}$$

and

$$u_*^2 = u_{*u}^2 + u_{*v}^2. \tag{46b}$$

In Equation (46a), z_{S0} is the prescribed roughness length of the substrate surface. As the lower boundary condition on TKE in conjunction with Equation (46), we adopted the equilibrium relationship $k_{\text{gnd}} = u_*^2/c_\epsilon$. An alternative prescription would be to impose $\partial k/\partial z = 0$ at $z = 0$.

The iteration procedure stops when the error (e.g., sum of absolute residuals or L1 norm, or root-mean-square residuals or L2 norm) in the four equations (viz., mean momentum and TKE balance equations) is less than a certain fraction of their initial error, or if a prescribed maximum number of iterations is exceeded (or, if the algorithm begins to show signs of divergence). SIMPLE ensures that the governing equations in their integral forms, which express the balance of sources within control volumes against the next flux across the control volume surface, are satisfied to within machine precision (in each such volume). As an additional convergence test, iterative refinement of all fields was continued until the integral form of the U-momentum equation, covering the entire flow domain, was satisfied to within 1% of the driving stress aloft.

V. AN EXAMPLE

A Fortran 95 program was developed to calculate the disturbed 3-D mean velocity field in an urban canopy using the methodology described in Section IV. An important program consistency check was conducted; namely, a test in which there is imposed no spatial inhomogeneity across the flow domain (building either does not exist, or uniformly occupies the horizontal level below $z = h_c$). For this test, one allows or forces the program to cycle in search of the 3-D flow solution, which ought to be identical to the 1-D solution used for the initialization. The initial (height-dependent only) fields should be sustained without degradation across the entire flow domain. This requirement has been verified several times for the current program implementation.

The first application of this program was to the calculation of disturbed flow within regularly arrayed roughness elements (buildings) immersed within a constant stress layer

that was used for the host flow For the simulation considered here, the imposed kinematic shear stresses aloft were prescribed as

$$-\overline{u'w'} = u_*^2, \qquad -\overline{v'w'} = 0, \tag{47}$$

where $u_* = 1$ m s^{-1}. The wind direction at the upper boundary was not prescribed (flux boundary conditions on U and V), but was in consequence of the stress specification in Equation (47) parallel to the x-direction. The Coriolis parameter f was set to zero for the simulation. Furthermore, the equilibrium ratios $c_u \equiv \sigma_u/u_{*0}$, $c_v \equiv \sigma_v/u_{*0}$, and $c_w \equiv \sigma_w/u_{*0}$ of the turbulent velocity standard deviations to the friction velocity for the reference (host) flow (i.e., neutral surface layer) were given the standard values $c_u = c_v = 2.2$ and $c_w = 1.25$, and imply a reference value for k_0/u_{*0}^2, and hence for the equilibrium constant c_e appearing the definition of the eddy viscosity K [cf. Equation (13)]. The constants c, α_*, and μ of the turbulence closure were given the same values as recommended by Wilson *et al.* [15] for vegetative canopies; namely, $c = \alpha_* = 1$ and $\mu = 0.2$.

The simulation consisted of an extensive (effectively infinite) array of cubical obstacles with a plan area density of $\lambda_p \equiv A_o/A_d = 0.25$, where A_o is the plan area of the cubical obstacles and A_d is the "lot" area (i.e., the ground surface area within which a single obstacle of the array sits). The arrangement of the obstacles was an in-line (square) format in which clear streets between the rows and columns of obstacles were evident. The dimensions of an obstacle was chosen to be $X = Y = h_c = 10$ m. Domain size was $L_x = L_y = 20$ m was chosen to ensure a plan area density of $\lambda_p = 0.25$. The top of the domain was chosen to be at $L_z = 80$ m. The grid resolutions in the x-, y-, and z-directions were chosen to be $\Delta x = \Delta y = \Delta z = 1$ m. The substrate roughness length z_{S0} was 0.1 m, but the simulations appear to be insensitive to this parameter provided $z_{S0} \ll h_c$. The drag parameter $C_d A = 10^3$ was purposefully chosen to be large in order to ensure that the cubical obstacles were essentially impermeable. Rather than use the center-of-pressure theorem [cf. Equation (16)] to determine the displacement length d required in the specification of the outer length scale λ_o [cf Equation (14a)], we used a similarity relationship for d for square arrays of cubes developed by Macdonald, Griffiths, and Hall [16]. In particular, these researchers found that for square arrays the displacement length can be determined from

$$\frac{d}{h_c} = 1 + A^{\lambda_p}(\lambda_p - 1), \tag{48}$$

where $A = 3.59$ is an empirical constant that has been determined by fitting the similarity

relationship to some experimental data. The program cycled through 5000 outer iterations, using nearly 3 hours of CPU time on a Pentium II PC with a clock speed of 400 MHz.

For now, one can only comment that the flow pattern calculated for the extensive in-line cubical obstacle array is at least *qualitatively* reasonable. Figure 1 displays vector plots of the calculated mean velocity field in a horizontal cross-section at $z/h_c = 0.5$ of the horizontal velocity vector (U, V). This figure displays the vector plots on both a linear (left-hand side) and logarithmic (right-hand side) scale. In this plot, one observes a "jet" of wind flow down the channel (street) that is unimpeded by the buildings, and the wind "swooshing" around the leeward corner of the building to form a standing vortex or eddy. It should be emphasized that this is an eddy in the mean flow. The program does not and cannot make predictions about the intermittent eddies. Figure 2 exhibits vector plots of the mean velocity components (U, V) in a horizontal cross-section at $z/h_c = 1.0$. Comparing Figure 2 with Figure 1, it is seen that the standing eddy does not occur higher up at $z/h_c = 1.0$. Figure 3 shows a vector plot of the mean velocity components (U, W) in a vertical cross-section [x-z plane at the lateral location $y = Y/2$ midway through the building]. The noticeable feature of this figure is the presence of the overall mean shear particularly at or near the urban canopy top (i.e., notice the longer velocity vectors at larger z). Furthermore, a standing leeward eddy can be seen in Figure 3. Incidentally, in this simulation, one is entitled to continue the same flow pattern by translation (not by reflection) on the outside of every lateral boundary. Continuation by reflection does not represent the lateral boundary conditions imposed in this simulation.

Figure 4 shows a horizontal profile along the x-direction of the local mean U-velocity through the building and across the street at the position $(y, z) = (4.5, 5.5)$ m in the flow domain. Note that a region of reverse flow occurs in the lee of the buildings as is also evident in the vector plots of Figure 1. The U-grid points do not fall on the building faces, and although evidently $U \approx 0$ well inside the building, a non-zero velocity occurs at the first interior grid point. This is explained by the spatial interpolation used by SIMPLE to estimate U at the boundary. Provided there are many grid points within the building (viz., high spatial resolution with $\Delta x \ll X$), an imperfect inside/outside building transition should carry no penalty. However, it should be emphasized that the advection (UU term in mean momentum budget equation) across the walls needs to be accounted for (in addition to the pressure forces and roof drag) in any global mean momentum macro-budget of the building-excluded domain.

The 3-D mean flow statistics in the urban canopy can be used to derive spatially averaged wind statistics. For this purpose, spatial averages in a horizontal plane were defined with building interior spaces excluded Figure 5 compares the displaced log-layer wind profile with the vertical profile of $< U > /u_{*h_c}$, where the canopy-top friction velocity was derived from the model output as

$$u_{*h_c} = \sqrt{|\langle \overline{u'w'} \rangle (z = h_c)|}. \tag{49}$$

The displaced log-layer wind profile

$$\frac{\langle U \rangle}{u_{*h_c}} = \frac{1}{k_v} \ln \left(\frac{z-d}{z_0} \right), \tag{50}$$

contains the two parameters d and z_0. The displacement length d was, of course, determined from the similarity relationship of Equation (48). For this example, Equation (48) gave $d/h_c = 0.46$. In view of this, a best effective urban layer roughness length, z_0, can now be obtained for the urban canopy by conducting a least-squares fit of the displaced log-layer wind profile to the computed model profile $\langle U \rangle / u_{*h_c}$ over the height range $2 \leq z/h_c \leq 5$. Note that the height range chosen for performing the fitting (to diagnose a value for z_0) implies an arbitrary guess as to the depth ranges of the roughness sub-layer, and the inertial sub-layer lying above it. Using the spatially averaged mean velocity profile shown in Figure 5, the roughness length for the squared urban array with $\lambda_p = 0.25$ was found to be $z_0/h_c = 0.089$.

VI. CONCLUSIONS

The development and numerical implementation of a 3-D model for the disturbed mean flow and turbulence in a regular urban canopy composed of a large group of obstacles has been described. Fixed bluff bodies exert a drag force on the incident atmospheric flow, so we proposed modelling an array of obstacles as a region of distributed drag force above the ground. The appeal of representing an individual building with a drag parameter $C_d A$ of "high enough" value as to render it as effectively solid is pure computational simplicity. By this simple device, one avoids the need for rendering a complex grid distribution about each building, and the explicit imposition of boundary conditions on the walls of all the buildings in the array. In so doing, a decision was made to ignore the small-scale flow details associated with the development of the viscous boundary layers around the walls and roof of

the buildings. Such details are forsaken, in the hope that they are less important than the large-scale building-induced circulations, as least as regards to any impact on the passive scalar dispersion.

The critical issue facing the current model is validation. There will be a pressing need to deal with the great volume and specificity of the model outputs from the current program, and to take these predictions beyond mere qualitative statements of interpretation such as "looks reasonable!" to more critical quantitative tests of the predictions. What would that test or series of tests be? As already mentioned, we do not seek to capture arcane flow details on individual buildings, so there is no point to turn to experiments on flow about single obstacles. With regard to this point, we have developed the current model to provide predictions of flow through large groups of buildings such as exist within a domain inside a large city. Unfortunately, we have not yet been able to identify an experimental study in which the flow within an extensive building array (viz., extensive enough to induce large-scale uniformity in the array and justify the assumption of periodicity inherent in the present model) is measured with comprehensive detail relative to separation length scales L_x and L_y. New laboratory experiments designed to provide these comprehensive flow details within an extensive urban canopy are currently being developed at Defence Research Establishment Suffield to obtain the necessary information to validate the current model. In the meantime, one might perhaps test the present model against the observations of Raupach *et al.* [17], who comprehensively described a wind tunnel simulation with a very long fetch over a staggered array of very thin obstacles (i.e., vertical bars or "tombstones"). However, this unusual canopy was designed more as an archetype for a plant canopy rather than an urban canopy.

Presently, a full 3-D computation of the disturbed flow field in an urban array requires computation times of the order of hours on a PC. In order for the model to be useful for operational applications, efficient algorithms will need to be developed that would enable numerical solutions for the coupled partial differential equations describing mean momentum and TKE transport to be obtained in computational times of the order of minutes. Certainly, with improvements in algorithms as well as in computer performance (CPU speeds, etc.), it is anticipated that this objective can be achieved within the next two years.

REFERENCES

1. Thomson, D. J., "Criteria for the Selection of Stochastic Models of Particle Trajectories in Turbulent Flows", J. Fluid Mech., 180, pp. 529–556, 1987.

2. Monin, A. S. and Yaglom, A. M., "Statistical Fluid Mechanics", The M.I.T. Press, USA, 1977.

3. Kaimal, J. C. and Finnigan, J. J., "Atmospheric Boundary-Layer Flows", Oxford University Press, UK, 1994.

4 Delage, Y., "A Numerical Study of the Nocturnal Atmospheric Boundary-Layer", Q. J. R. Met. Soc., 100, pp. 351–364, 1974

5. Raupach, M. R., Finnigan, J. J., and Brunet, Y., "Coherent Eddies and Turbulence in Vegetation Canopies: The Mixing-Layer Analogy", Boundary-Layer Meteorol., 78, pp. 351–382, 1996.

6. Jackson, P. S., "On the Displacement Height in the Logarithmic Velocity Profile", J. Fluid Mech., 111, pp. 15–25, 1981.

7. Launder, B. E., Reece, G. T., and Rodi, W., "The Development of a Reynolds Stress Turbulent Closure", J. Fluid Mech., 68, pp. 537–566, 1975.

8. Rodi, W., "Turbulence Models and Their Application in Hydraulics", IAHR, The Netherlands, 1984.

9. Pope, S. B., "A More General Effective Viscosity Hypothesis", J. Fluid Mech., 72, pp. 331–340, 1975.

10. Gatski, T. B. and Speziale, C. G., "On Explicit Algebraic Stress Models for Complex Turbulent Flows", J. Fluid Mech., 254, pp. 59–78, 1993

11. Wolfram, S., "Mathematica", Addison-Wesley, USA, 1988.

12. Spencer, A. J. M., "Theory of Invariants", in *Continuum Physics* (ed. A. C. Eringen), Vol. 1, Academic Press, USA, pp. 240–352, 1971.

13. Patankar, S.V., "Numerical Heat Transfer and Fluid Flow", Hemisphere Publishing Corporation, New York, 1980.

14. Van Doormal, J. P. and Raithby, G. D., "Enhancements of the SIMPLE Method for Predicting Incompressible Fluid Flows", Numer. Heat Transfer, 7, pp. 147–163, 1984.

15. Wilson, J. D., Finnigan, J. J., and Raupach, M. R., "A First-Order Closure for Disturbed Plant-Canopy Flows, and Its Applications to Winds in a Canopy on a Ridge", Q. J. R. Meteorol. Soc., 124, pp. 705–732, 1998.

16. Macdonald, R. W., Griffiths, R. F., and Hall, D. J., "An Improved Method for the Estimation of Surface Roughness of Obstacle Arrays", Atmospheric Environment, 32, pp. 1857–1864, 1998.

17. Raupach, M. R., Coppin, P. A., and Legg, B. J., "Experiments on Scalar Dispersion Within a Model Plant Canopy, Part I: The Turbulence Structure", Boundary-Layer Meteorol., 35, pp. 21-52, 1986.

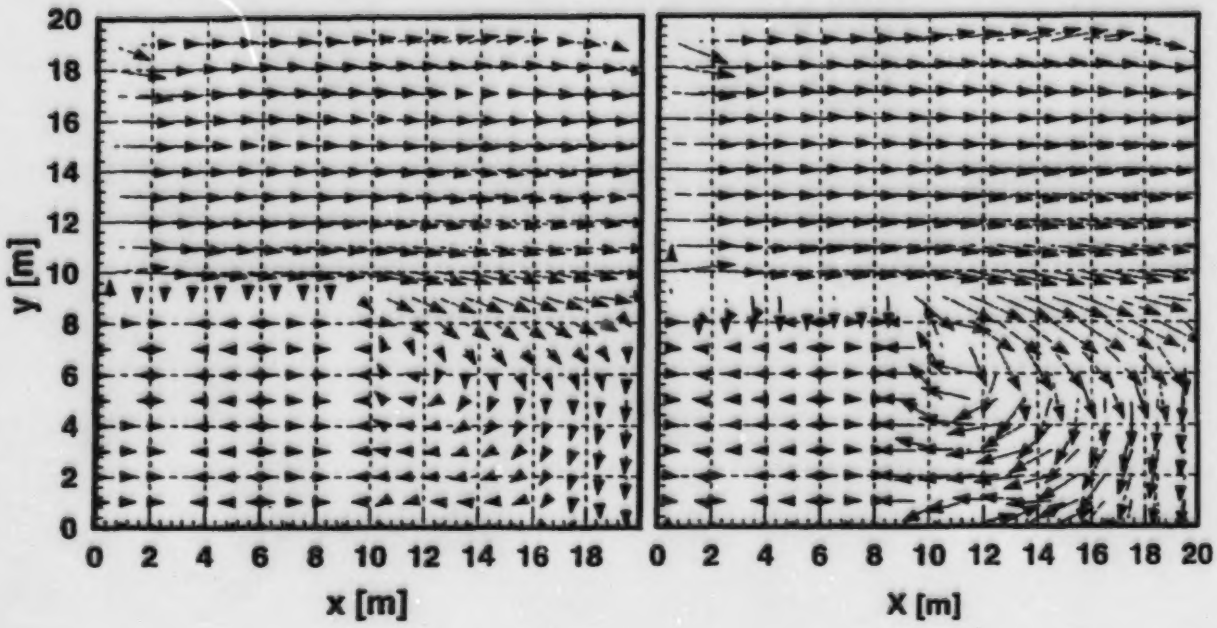

Figure 1. Calculated spatial field at $z/h_c = 0.5$ of the horizontal velocity vector (U,V), for wind flow about a square array of cubical roughness elements ($X = Y = h_c = 10$ m) within a constant stress layer flow. Computational domain size $L_x = L_y = 20$ m. The plan area density is 0.25. The plot on the left uses a linear scale and the plot on the right uses a logarithmic scale for the length of the wind vectors.

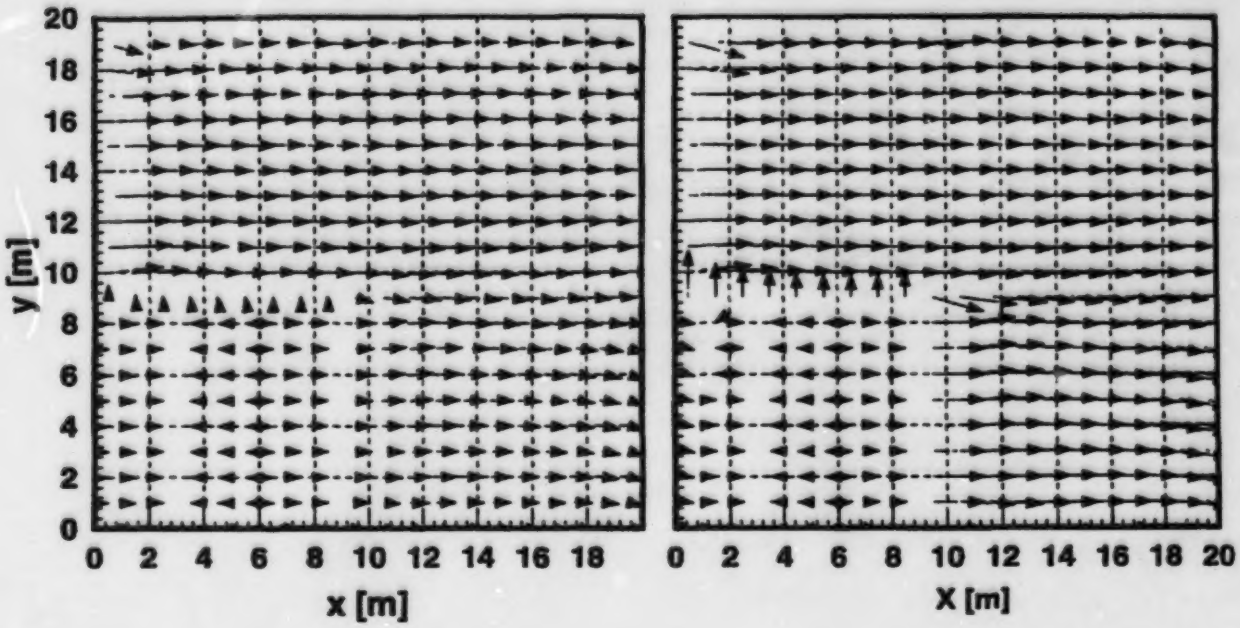

Figure 2. Calculated spatial field at $z/h_c = 1.0$ of the horizontal velocity vector (U,V), for wind flow about a square array of cubical roughness elements ($X = Y = h_c = 10$ m) within a constant stress layer flow. Computational domain size $L_x = L_y = 20$ m. The plan area density is 0.25. The plot on the left uses a linear scale and the plot on the right uses a logarithmic scale for the length of the wind vectors.

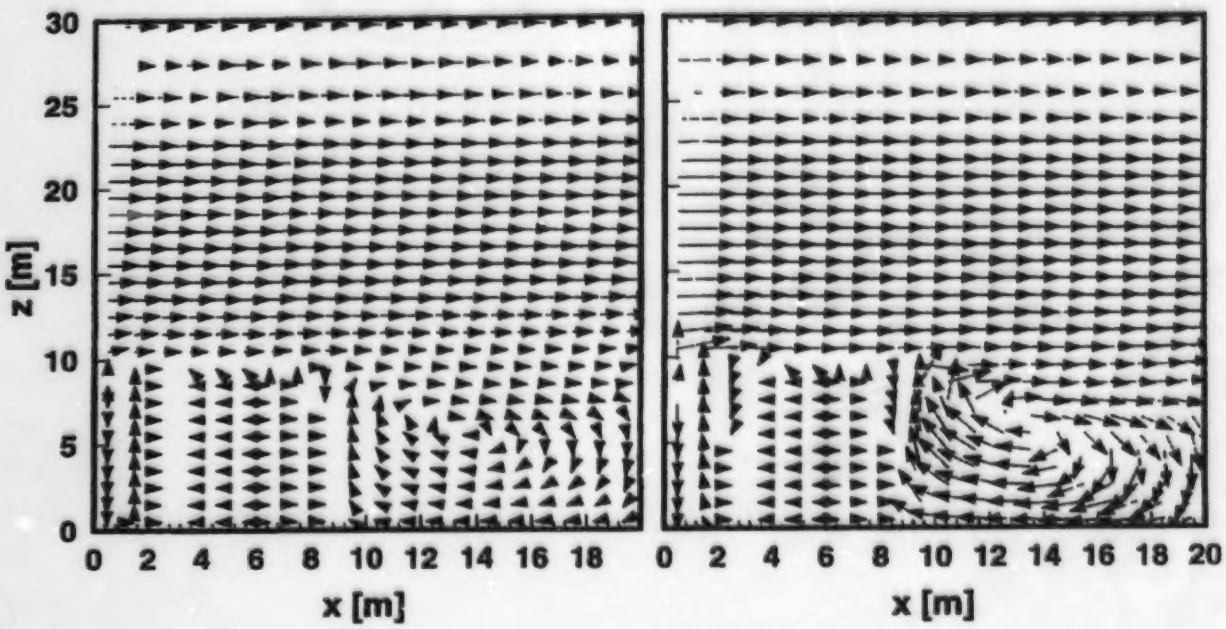

Figure 3. Calculated spatial field at $y/Y = 0.5$ of the velocity vector (U,W), for wind flow about a square array of cubical roughness elements ($X = Y = h_c = 10$ m) within a constant stress layer flow. Computational domain size $L_x = L_y = 20$ m. The plan area density is 0.25. The plot on the left uses a linear scale and the plot on the right uses a logarithmic scale for the length of the wind vectors.

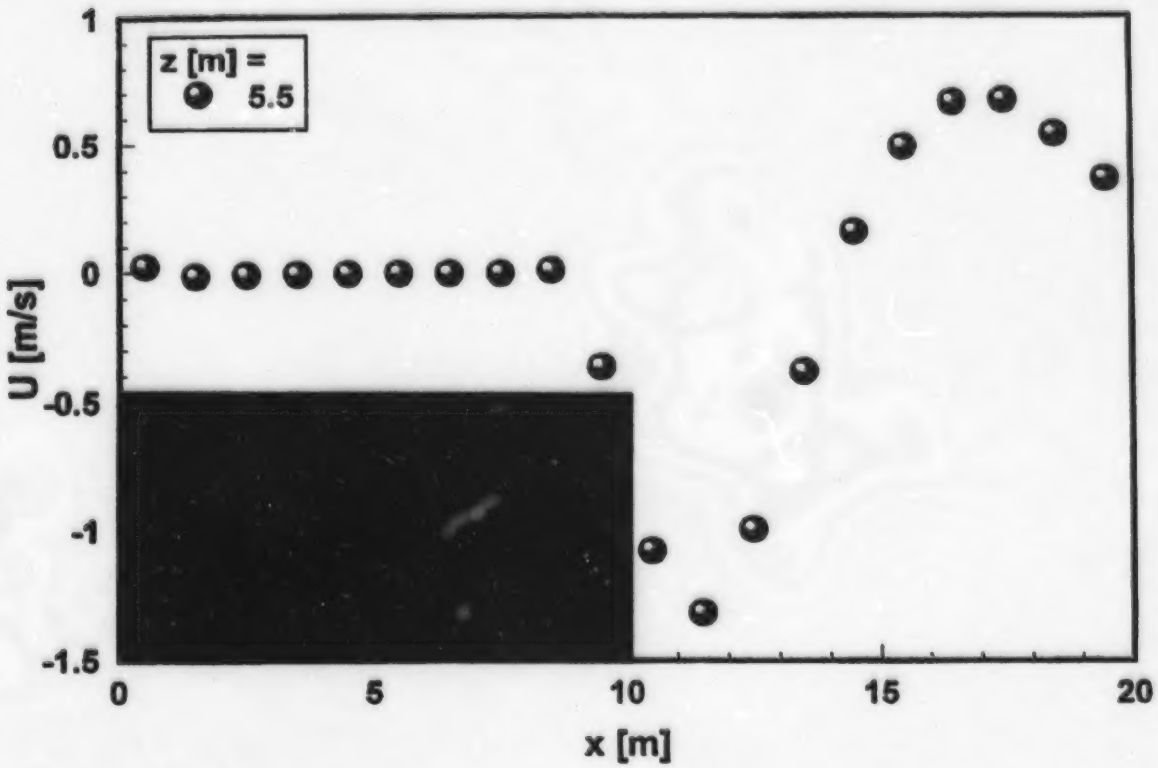

Figure 4. Local profile of $U(x,y,z)$ versus alongwind coordinate x through the building at $(y,z) = (4.5,5.5)$ m.

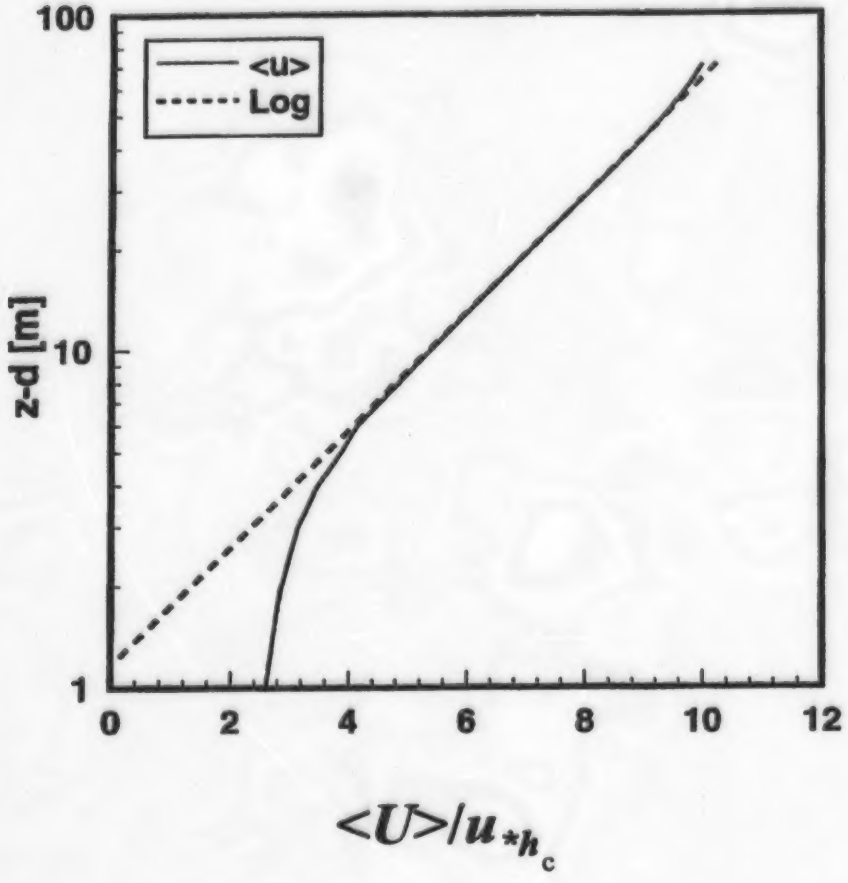

Figure 5. Vertical profile of the normalized spatially averaged velocity <*U*> as calculated by the flow model. Also shown, the displaced log-law wind profile with roughness length, z_0, optimized for least squares best fit in the inertial sub-layer.

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In this paper, a three-dimensional (3-D) model for the disturbed mean wind flow and turbulence in an urban canopy composed of an extensive array of obstacles is developed. The context of this work is the provision of a computational tool for the rapid diagnosis of transport and dispersion of toxic contaminants (e.g , chemical and biological warfare agents) over a relatively homogeneous urban area. A variant of the $K \sim \lambda k^{1/2}$ first-order closure (where K is the eddy viscosity, k is the turbulence kinetic energy (TKE) calculated from its simplified governing differential equation, and λ is an algebraically specified turbulence length scale) is studied and used to close the mean momentum equations and the TKE transport equation The turbulence length scale λ has been designed to reproduce the known characteristics of the length scale in a canopy; namely, recognition of the limitation imposed on eddy size by proximity to the ground, the constant length scale across much of the upper canopy that is intimately connected to the canopy shear length scale (vorticity thickness), and the displacement of the equilibrium flow structure above the canopy due to the presence of the canopy. A method for the partitioning of the TKE into its component normal Reynolds stresses is obtained. The latter is based on the derivation of a particular algebraic stress model that has been derived from a stress transport model due to Launder, Reece and Rodi (Journal of Fluid Mechanics, 68, pp. 537--566, 1975) that has been modified to include the extra physics arising from the interaction of the airflow with the obstacles. The effect of the obstacles on the flow is parameterized in terms of an effective drag parameter, thus foregoing the complications involved in the generation of a body-fitted grid about the obstacles and imposing no slip/no leak boundary conditions at the walls of the obstacles. Details of the numerical implementation of this model are provided. An example of a simulation of the disturbed mean flow in an urban array is given.

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